A Robust Implementation of an Algorithm to Solve the S-unit Equation

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A simple equation

\[ x + y = 1 \]
Applications of the $S$-unit equation

- Classification of genus 2 curves
- An asymptotic version of Fermat's Last Theorem
- Ramanujan–Nagell equations
Tools Used to Solve the $S$-unit equation

- $\mathbb{Q}$-linear combinations of logarithms (archimedean and not)
- LLL-algorithm (Lenstra, Lenstra, Lovász)
- Sieving and/or Brute Force
What are $S$-integers?

<table>
<thead>
<tr>
<th>Field</th>
<th>$\mathbb{Q}$</th>
<th>$K/\mathbb{Q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primes</td>
<td>$p$</td>
<td>$\mathfrak{p}$</td>
</tr>
<tr>
<td>Integers</td>
<td>$\mathbb{Z}$</td>
<td>$\mathcal{O}_K$</td>
</tr>
<tr>
<td>Units</td>
<td>${\pm 1}$</td>
<td>$\omega_K \times \mathbb{Z}^r$</td>
</tr>
<tr>
<td>Valuations</td>
<td>$v_p(\alpha)$</td>
<td>$v_\mathfrak{p}(\alpha)$</td>
</tr>
<tr>
<td>Absolute Values</td>
<td>$</td>
<td>\cdot</td>
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</tbody>
</table>

(a) $K = \mathbb{Q}$, $\mathbb{Z} = \{\alpha \in \mathbb{Q} : v_p(\alpha) \geq 0 \ \forall \ p \ \text{prime}\}$

(b) $K/\mathbb{Q}$, $\mathcal{O}_K = \{\alpha \in K : v_\mathfrak{p}(\alpha) \geq 0 \ \forall \ \mathfrak{p} \ \text{prime ideal}\}$