AN EULER PHI FUNCTION FOR THE EISENSTEIN INTEGERS
AND SOME TANTALIZING APPLICATIONS

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ABSTRACT:

The Euler phi function on a given integer $n$ yields the number of positive integers less than and relatively prime to $n$. Equivalently, it gives the order of the group of units in the quotient ring $\mathbb{Z}/(n)$ for a given integer $n$. We generalize the Euler phi function to the Eisenstein integer ring $\mathbb{Z}[\rho]$ where $\rho$ is the primitive third root of unity $e^{2\pi i/3}$ by finding the order of the group of units in the ring $\mathbb{Z}[\rho]/(\theta)$ for any given Eisenstein integer $\theta$. As one application, we prove that the celebrated Euler-Fermat theorem holds for the Eisenstein integers. We also discuss the structure of certain unit groups $(\mathbb{Z}[\rho]/(\gamma^n))^\times$ where $\gamma$ is prime in $\mathbb{Z}[\rho]$ and $n \in \mathbb{N}$, thereby generalizing well-known results of similar applications in the integers and some lesser known results in the Gaussian integers.