Combinatorial consequences of the representation stability of the cohomology of Springer varieties

Aba Mbirika of UW-Eau Claire

Abstract

A sequence of $S_n$-representations $\{V_n\}$ is said to be uniformly representation stable if the decomposition of $V_n = \bigoplus \lambda c_{\lambda,n} V(\lambda)_n$ into irreducible representations can be described independently of $n$ for each $\lambda$—that is, the multiplicities $c_{\lambda,n}$ are eventually independent of $n$ for each $\lambda$. It is known that uniform representation stability holds for the cohomology of flag varieties (the so-called diagonal coinvariant algebra), a well-known Springer representation. We prove that this also holds for every sequence of Springer representations. In this talk we only sketch the proof of how we exhibited the co-FI-module structure (in the sense of Church-Ellenberg-Farb) of a sequence of cohomology rings of Springer varieties parametrized by partitions of $n$. For the majority of the talk, we explore some combinatorial consequences of this stability of the Springer representation. This will include some conjectures that we have yet to prove but will provide convincing evidence that they indeed hold.