Symmetry of graphs. Circles
What is Symmetry?

Take some geometrical object. It is called symmetric if some geometric move preserves it.
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**Examples with Symmetry**

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- Reflection across $y$ sends $(x, y)$ to $(-x, y)$.
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Examples with Symmetry

![Graph](image)

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Examples with Symmetry
Reflect a point:

Reflection across the $x$–axis is given by $(x, y) \mapsto (x, -y)$

Reflection across the $y$–axis is given by $(x, y) \mapsto (-x, y)$

Reflection across the origin is given by $(x, y) \mapsto (-x, -y)$

Example:
For the point $P = (4, -2)$, write down the coordinates of and draw
  
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Testing for symmetry for graphs of equations

A graph is called **Symmetric** with respect to a reflection if that reflection does not change the graph.

The following graphs are symmetric about (a) reflection across the $x$-axis (b) reflection across the $y$-axis (c) reflection about the origin.

\[
\begin{align*}
  y &= x^2 - 4 \\
  y &= x^3 - 3x \\
  x &= y^2 + 1 \\
  x^2 + y^2 &= 4
\end{align*}
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**Analytic symmetry:**
The graph of an equation is symmetric about the $y$-axis if you can replace $x$ by $-x$ and get an equivalent equation.
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Check these examples.
checking for symmetry analytically

What Symmetries does the graph of $x^3 + x = y^2 - 1$ have?

- Across the $x$-axis?

Replace $y$ by $-y$ to get

$$x^3 + x = (-y)^2 - 1$$

which is equivalent to

$$x^3 + x = y^2 - 1$$

This graph is Symmetric about $x$-axis.

- Across the $y$-axis?

Replace $x$ by $-x$ to get

$$(-x)^3 + (-x) = y^2 - 1$$

which is equivalent to

$$-x^3 - x = y^2 - 1$$

This is not equivalent to the original equation. This graph is not symmetric about $x$-axis.

Symmetric about the origin?

Replace $x$ by $-x$ and $y$ by $-y$ and see if you get an equivalent equation.

I'll pause the lecture here to mention Wolfram alpha.
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- Symmetric about the origin?
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Using Symmetry to graph

We can use symmetry to cut the work of graphing in half.

Without a computer graph \( y = 4 - x^2 \):

\[
\begin{align*}
\text{Evaluate at } x &= 0, 1, 2, 3, 4; \\
\text{Symmetric about the } y\text{-axis} \\
\text{We only need to check positive } x\text{'s: The negatives will follow from symmetry.}
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Without a computer graph \( y = \frac{1}{x} \):

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\text{Evaluate at } x &= 1/4, 1/3, 1/2, 1, 2, 3, 4; \\
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Let's use Wolfram \( \alpha \) (alpha) to check our work.
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Equations for Circles

The circle centered at \((h, k)\) with radius \(r\) is

- The set of all points \((x, y)\) which are exactly \(r\) away from \((h, k)\)

The standard equation for the circle of radius \(r\) centered at \((h, k)\) is

\[
(x - h)^2 + (y - k)^2 = r^2
\]

1. Give the standard equation for the circle centered at \((0, 1)\) of radius 2.

2. What are the center and radius of the circle given by

\[
(x - 3)^2 + (y - 5)^2 = 9
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Equations for Circles

The circle centered at \((h, k)\) with radius \(r\) is

- The set of all points \((x, y)\) which are exactly \(r\) away from \((h, k)\)

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Give the standard equation for the circle centered at \((0, 1)\) of radius 2.

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Graph it
Equations for Circles

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Equations for Circles

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Equations for Circles

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Finding the $x$ and $y$-intercepts of circles

The standard equation for the circle of radius $r$ centered at $(h, k)$ is

$$(x - h)^2 + (y - k)^2 = r^2$$

Find the $x$-intercepts of the circle centered at $(7, 3)$ with radius 5.

Plug $y = 0$ into $(x - 7)^2 + (0 - 3)^2 = 25$.

$(x - 7)^2 + 9 = 25$

$(x - 7)^2 = 16$

$x - 7 = \pm 4$

$x = 7 \pm 4$

Two $x$-intercepts: One at $(11, 0)$ one at $(3, 0)$.

Find the $y$-intercepts by setting $x = 0$. Are there any?
Finding the x and y-intercepts of circles

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Symmetry of graphs. Circles
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Does the equation \( x^2 + 2x + y^2 - 4y = 20 \) give a circle?
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Let’s try a wolfram alpha plot.

Let’s review Completing the square.

Expand $(x+1)^2 = x^2 + 2x + 1$.
So, $x^2 + 2x = (x+1)^2 - 1$.

Expand $(y-2)^2 = y^2 - 4y + 4$.
So, $y^2 - 4y = (y-2)^2 - 4$.

Substituting these into our original equation we get:

$$(x+1)^2 - 1 + (y-2)^2 - 4 = 20$$

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The equation for the circle of radius 5 centered at $(-1, 2)$.
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Completing the square and the general equation for a circle.

Proposition

The completing the square formula says that:

\[ x^2 + bx = \left( x + \frac{b}{2} \right)^2 - \frac{b^2}{4} \]

Complete the relevant squares to determine the radii (The plural of radius) and centers of the circles with equations

- \( x^2 + 4x + y^2 − 8y = 24 \)
- \( x^2 + x + y^2 − 3y = 5 \)

Homework:
F.2: 77, 78, 79, 80, 84, 85, 86
F.4: 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 21, 22, 27, 28,