Direct Method of Interpolation

What is Interpolation?

- Estimation of intermediate values between precise data points. The most common method is:

\[ f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n \]

- Although there is one and only one \( n \)-th order polynomial that fits \( n+1 \) points, there are a variety of mathematical formats in which this polynomial can be expressed:
  - The Newton polynomial
  - The Lagrange polynomial

What is Interpolation?

Given \((x_0,y_0), (x_1,y_1), \ldots, (x_n,y_n)\), find the value of ‘y’ at a value of ‘x’ that is not given.

Interpolants

\[ f(x_i) = y_i, \quad i = 1, \ldots, n \]

A pair \((x,y)\) is called data point and the function \(f\) is called the interpolant for the data points

Polynomials are the most common choice of interpolants because they are easy to:

- Evaluate
- Differentiate, and
- Integrate.
Interpolants
Examples of data points

Newton’s Divided-Difference Interpolating Polynomials

Linear Interpolation
• Is the simplest form of interpolation, connecting two data points with a straight line.

\[
\frac{f(x) - f(x_0)}{x - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}
\]

Linear-interpolation formula
• \( f_1(x) \) designates that this is a first-order interpolating polynomial.

Example
The upward velocity of a rocket is given as a function of time in Table 1. Find the velocity at \( t = 16 \) seconds using the Newton Divided Difference method for linear interpolation.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( v(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>227.86</td>
</tr>
<tr>
<td>15</td>
<td>362.78</td>
</tr>
<tr>
<td>20</td>
<td>517.35</td>
</tr>
<tr>
<td>22.5</td>
<td>602.97</td>
</tr>
<tr>
<td>30</td>
<td>901.67</td>
</tr>
</tbody>
</table>

Table: Velocity as a function of time

Linear Interpolation
\[
v(t) = b_0 + b_1 (t - t_0)
\]

\( t_0 = 15, v(t_0) = 362.78 \)
\( t_1 = 20, v(t_1) = 517.35 \)
\( b_0 = v(t_0) = 362.78 \)
\( b_1 = \frac{v(t_1) - v(t_0)}{t_1 - t_0} = 30.914 \)
Linear Interpolation (contd)

\[ v(t) = b_0 + b_1(t - t_0) \]

At \( t = 16 \)
\[ v(16) = 362.78 + 30.914(16 - 15) \]
\[ v(16) = 393.69 \text{ m/s} \]

Quadratic Interpolation

- If three data points are available, the estimate is improved by introducing some curvature into the line connecting the points.

\[ f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) \]

- A simple procedure can be used to determine the values of the coefficients.

\[ x = x_0 \]
\[ b_0 = f(x_0) \]
\[ x = x_1 \]
\[ b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \]
\[ x = x_2 \]
\[ b_2 = \frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0} \]

General Form of Newton’s Interpolating Polynomials

\[ f_n(x) = f(x_0) + (x - x_0)f[x_1, x_0] + (x - x_0)(x - x_1)f[x_2, x_1, x_0] \]
\[ + \cdots + (x - x_0)(x - x_1)\cdots(x - x_{n-1})f[x_n, x_{n-1}, \ldots, x_0] \]
\[ \text{Bracketed function evaluations are finite divided differences} \]

Problems
Spline Interpolation

- There are cases where polynomials can lead to erroneous results because of round off error and overshoot.
- Alternative approach is to apply lower-order polynomials to subsets of data points. Such connecting polynomials are called spline functions.

Examples