The theory of fiat money and private money as alternative media of exchange

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ABSTRACT

A random-matching model with a clearinghouse is constructed to investigate the impact of private money on economic efficiency and social welfare in three monetary regimes. A subset of agents, called bankers, whose credit histories are recorded by the clearinghouse, are allowed to issue private banknotes in order to consume. Those private liabilities may serve as media of exchange, either by themselves, or alongside a stock of fiat money. Under certain conditions, welfare in a monetary steady state with private money is strictly higher than that attained in a steady state where private money is prohibited.

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1. Introduction

One of the ongoing controversies in monetary economics is whether the private sector should be allowed to create money. The primary debates were between what were called the banking and the currency schools, with the former (Hayek, 1976) advocating laissez-faire in intermediation and the latter (Friedman, 1960) advocating a complete government monopoly over currency issuance. Somewhere between these views lies the real bills doctrine, which favors the coexistence of publicly and privately-issued circulating liabilities. These three points of view form the core of the most received theory on private money.

In the past seventy years, implicit restrictions, such as prohibitively high tax on banknotes have prevented the private issue of money in the United States. Recent legislative developments, however, have removed those impediments. At the same time, a variety of “e-cash”, the electronic equivalents of private banknotes have appeared due to advances in communication and transaction technologies. Therefore, we can plausibly expect to see a return to a situation where fiat money and private notes circulate alongside each other. This fact suggests that studying the above three viewpoints is relevant and important.

The last decade has witnessed dramatic developments in the theoretical understanding of private money. Related work includes Kiyotaki and Wright (1993), Aiyagari et al. (1996), Smith and Weber (1999), Azariadis et al. (2001), Temzelides and Williamson (2001), Bullard and Smith (2003) and so on. Particularly, Cavalcanti and Wallace (1999a,b) formulate a random-matching model to investigate private note issue and redemption. By examining two exclusive cases, “fiat money only” and “private money only”, they show that the set of implementable allocations using fiat money is a strict subset of the set using private money. Cavalcanti, Erosa, and Temzelides (1999) build a model to understand private note exchange, where banking is made possible by the introduction of a clearinghouse. Focusing on monetary stability, they find conditions under which note redemptions can discipline note issue by the banking sector. Our paper is greatly inspired by the former two models. In contrast to previous work, our model goes further by characterizing equilibria.
and investigating the welfare implications of private notes. Particularly, we incorporate into our framework the monetary regime where fiat money and private banknotes coexist. In this paper, a random-matching model with a clearinghouse is set out to study the performance of economies, where three monetary regimes are considered. An arrangement that resembles private banknote issue can be compared with arrangements that resemble government monopoly on money issue and a mix of government and private money issue, respectively. Then we try to examine which monetary regime is most preferable in welfare terms, “fiat money only”, “private money only”, or “coexistence of fiat money and private money”.

Martin and Schreft (2006) establish the existence of equilibria with competitive issue of fiat money in both a search and an OG framework. However, it is ambiguous whether the equilibria with competitive issuers have more desirable welfare properties. Our analysis shows that the welfare obtained in the monetary regime with private money (competitively issued medium of exchange) is generally higher than what is achieved in a regime where private money is not allowed.

Our attention is confined to steady states throughout the paper. First, we study a case with no banking sector and with an exogenous amount of indivisible fiat money, where each agent’s trading history is assumed to be private information. Therefore, fiat money must be used to overcome the incentive problem in bilateral transactions. Fiat money is then valued as a means of payment in the unique monetary equilibrium. Then we get rid of fiat money and introduce a banking sector with a clearinghouse. A subset of agents, called bankers, are allowed to keep reserves in the clearinghouse. The treatment of reserves here acts as a gathering of trading history. Banks are able to issue indivisible banknotes for consumption. These privately-issued notes circulate as potential media of exchange and can be returned to banks for redemption. Our results show that under certain conditions, a unique stationary equilibrium exists in this regime. Next, we consider a situation where fiat money and private notes coexist. In this case, fiat money not only serves as a medium of exchange but also can be directly used to meet the reserve rule, while private notes function the same way as before. Due to a strategic complementarity, there exist two monetary steady states, which can be welfare-ranked. Finally, numerical examples are employed to demonstrate the corresponding welfare across three regimes.

The model also implies that the introduction of fiat money sometimes might be harmful to the economy. For example, in the regime of “fiat money only”, money cannot be returned to the (single) issuer for redemption. This “inconvertibility” is likely to induce the government to increase the stock of money at will and a higher price results. Also, the circulation of fiat money in the regime of “coexistence” leads to multiple equilibria which dominate each other in terms of welfare and therefore, represents a coordination problem.

The remainder of the paper is organized as follows. In Section 2, the basic model is built. In Section 4, we give a simple analysis shows that the welfare obtained in the monetary regime with private money (competitively issued medium of exchange) is generally higher than what is achieved in a regime where private money is not allowed.

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The remainder of the paper is organized as follows. In Section 2, the basic model is built. In Section 4, we give a simple introduction to the clearinghouse. In Sections 3, 5 and 6, we explore the economic implications of three different monetary regimes, which vary according to the asset circulating in the economy. Section 7 is a conclusion.

2. The basic model

Time is discrete and there is a continuum of infinite-lived agents with unit mass. Following Williamson (1999), we assume each agent has preferences given by

\[ E_0 \sum_{t=0}^{\infty} \beta^t [\theta_t u(c_t) - x_t] \]

(1)

where \( E_0 \) is the expectation operator, conditional on information available to the agent at date 0, \( \beta > 0 \) is the discount rate, \( c_t \) is the consumption in period \( t \), \( x_t \) is production of the consumption good. Let \( r \) denote the rate of time preference, where \( \beta = \frac{1}{1+r} \). As well, \( \theta_t \in [0, 1] \) is an i.i.d. preference shock over time and across agents, with \( \text{Pr}[\theta_t = 1] = \frac{1}{2} \). We assume that \( u(\cdot) \) is strictly increasing and strictly concave with \( u'(0) = 0 \) and \( u''(0) = \omega \). There is some \( q \geq 0 \) such that \( u(q) = 0 \). An agent’s own output can only be consumed by other agents. These goods are otherwise perishable and thus they cannot be used as commodity money.

In every period, there is no central market place to exchange and agents are randomly assigned to meet bilaterally. Trade can only occur between an agent who wishes to consume \( (\theta_t = 1) \) and an agent who doesn’t want to consume \( (\theta_t = 0) \), which rules out double-coincidence-of-wants matches. This generates a role for media of exchange. Each agent has only one chance to trade during each period. In this model, there will be three relevant cases to consider, which differ according to the asset circulating in the economy.

3. Fiat money only

There exists in the economy one intrinsically useless, indivisible and durable object called fiat money. For tractability, an upper bound on money inventory is imposed: Individuals can hold no more than 1 unit of monetary assets. We assume that a fraction \( \mu_m \) (\( 0 < \mu_m < 1 \)) of the agents start their lives each with one unit of fiat money, which can not be returned to the issuer (government) for redemption. In all random pairwise meetings, an agent does not have access to the histories of the other agent. That is, the private trading history of each agent helps preclude any form of credit in this economy. Money is essential in facilitating bilateral trades.

Now, suppose that two agents match and there exists a single coincidence. In this case, the seller supplies \( q_m \) units of goods in exchange for 1 unit of money from the buyer. To determine \( q_m \), we assume that buyers make take-it-or-leave-it offers to sellers, so that sellers get zero surplus from the trading while buyers get all the surplus. That is

\[ -q_m + V_m^n - V_0^n = 0 \]

(2)

\[ u(q_m) + V_0^n - V_m^n \geq 0 \]

(3)
where $V_0$ is the value of being a non-banker seller (producer) at the end of a period, $V_m$ is the value of being a non-banker money holder. From condition (2), we have $q_m = V_m - V_0 = V_m - V_0$.

In this paper, we restrict our attention to symmetric monetary steady states and derive the conditions under which fiat money is valued. Let $\rho_0$ denote the fraction of agents without money and similarly $\rho_m$ be the fraction of agents holding money in steady state. It is clear that

$$\rho_0 + \rho_m = 1, \quad \text{where} \quad \rho_m = \mu_m \quad (4)$$

The Bellman equations imply

$$rV_m^m = \frac{\rho_0^m[u(q_m) + V_0 - V_m^m]}{2} \quad (5)$$

$$rV_0^m = \frac{\rho_m^m(-q_m + V_m^m - V_0^m)}{2} \quad (6)$$

**Definition 1.** A monetary steady-state equilibrium for the economy without banking is a vector $[V_m^m, V_0^m, \rho_m^m, \rho_0^m]$ and a quantity $q_m > 0$ satisfying Eqs. (2)–(6).

**Proposition 1.** A unique monetary steady-state equilibrium exists if $\rho_m^m \in (0, 1)$ and the discount factor $\beta$ is sufficiently close to one.

### 4. Introduction of the clearinghouse

Historically, some monetary systems with circulating private notes had arrangements for clearing notes and reserves, but some did not. For example, between 1825 and the Civil War, the bulk of the U.S. money supply consisted of notes issued by private banks. Generally, those bank notes were redeemable at par on demand only in the location of issuance, while notes circulating sufficiently far away from their origin often exchanged at a discount. However, in New England during this period, a portion of bank notes circulated at par within the areas covered by the Suffolk Banking System, which opted to allow all system members to deposit with it any notes issued by any system members at par and also established the first net-clearing system in the U.S. to deal with the notes deposited. Due to its note acceptance and clearing strategy, the Suffolk Banking System achieved a generalized par circulation of their notes.

To facilitate the banknote circulation and redemption in the model, we now introduce a record-keeping technology operated by the private sector, called a clearinghouse, which keeps a reserve balance for each member. We will refer to the members as banks, and to the non-members as non-banks. The treatment of reserves here acts as a gathering of trading history that entails a credit arrangement for banks. This implies the clearinghouse has no public record for non-banks. Member banks can deposit reserves with the clearinghouse by recording the acceptance of monetary assets in trade meetings and reserve transference among banks. When bank $i$ produces goods for somebody in exchange for fiat money, the reserve balance of bank $i$ is credited by one unit at the end of the date. The deposited money is destroyed. If the deposited asset is a private note issued by another bank $j$ ($j \neq i$), the clearinghouse would credit bank $i$'s balance by one and meanwhile bank $j$’s balance would be debited by one. This process corresponds to note redemption, which could be implemented by bankers anywhere at any time.

Given that there is a continuum of agents, a bank will not meet its own issued note during the redemption process. Also, bankers in a particular state are allowed to convert central reserves into fiat money and use it to purchase consumption goods. Their reserve balance at the clearinghouse will be debited by one unit only if they have spent that money. At the end of each date, the clearinghouse will implement reserve settlement and update the reserve balance of all banks. Note, for tractability, we shall keep the tradition of restricting the reserve balance of an individual banker in the economy to either 0 or 1 unit. Central reserves are assumed to be indivisible.

In this model, bankers are permitted to issue notes in the absence of reserves. As we know, there is usually a time lag between note issue and note redemption, therefore the assumed “reserve rule” in this model corresponds to a regulation that is much weaker than 100% reserve requirements.

### 5. Private money only

Inspired by Cavalcanti and Wallace (1999a,b) and Cavalcanti et al. (1999), we assume a portion of the agents are now endowed with a technology to issue private money and to keep reserves with a clearinghouse. They are the so-called banks, whose credit histories are recorded and revealed through the clearinghouse. We will show there exist some equilibrium supported by private money that has a higher welfare than any supported solely by fiat money.

#### 5.1. Economic environment

At the initial date, a fraction $\mu_b$ ($0 < \mu_b < 1$) of agents start with the clearinghouse membership privileges and act as bankers. The remaining agents are non-bankers. To keep matters relatively simple, we assume that there is no fiat money in transactions.
Meanwhile, a fraction $\phi$ of bankers ($\phi \cdot \mu_b$) is randomly chosen to have 1 unit of indivisible reserve deposit with the clearinghouse in terms of some outside money. Bankers are identical to non-bankers regarding all the physical elements so far presented, except that bankers are able to create their own personal currencies—indivisible and perfectly durable objects, called banknotes—at no cost. Banknote would potentially circulate as a medium of exchange in this environment. Assume that the quantity of outstanding banknotes for each banker can only be 0 or 1. That is, a banker can not issue a new note unless the old one has been redeemed.

The sequence of actions within a period occurs as follows. Each agent starts with a state. The state for a non-banker is denoted by note holdings, a member of the set $\{0, 1\}$. The state for a banker is represented as $(n, \bar{r})$, where $n$ is the quantity of her previously-issued note that is still in circulation and $\bar{r}$ is the reserve balance. Note, both $n$ and $\bar{r}$ are also a member of the set $\{0, 1\}$. At the same time, the preference shock is revealed such that each agent knows whether she needs to consume or not. Then agents could contact the clearinghouse costlessly to acquire the information about other bankers’ reserve balances, the number of outstanding banknotes and so on.

Next, agents meet pairwise and at random. Bankers in particular states are permitted to issue private notes for goods. Holders of banknotes can purchase goods from would-be producers or go to a bank for redemption. At the end of a period, the clearinghouse accomplishes the reserve-clearing. Then a new period begins. We impose symmetry across agents and assume the non-bank public could contact the clearinghouse costlessly to acquire the information about other bankers’ reserve balances, the number of outstanding banknotes and so on.

In Eq. (9), the second term describes the situation where an agent with a banknote meets a banker whose reserve balance is 0 with probability $\rho^b_{1,0}$, who gives the agent a note in exchange for consumption. Assume that the quantity of outstanding banknotes and so on.

In Eq. (9), the second term describes the situation where an agent with a banknote meets a banker whose reserve balance is 0 with probability $\rho^b_{1,0}$. This process is called note redemption, which could be implemented conditional on the original issuer has 1 unit of reserve with the clearinghouse. That is, with probability $\rho^b_{1,0}$, the note redemption requirement would be honored. During this process, the note holder hands out her banknote and receives $R^b$ units of goods for consumption, where the redemption value $R^b$ is dependent on the state of the redeeming bank. The necessary condition for trades to occur is the net gain from trade is non-negative. If $R^b < 0$, this would-be-producer meets with a banker who wishes to consume and is capable to issue a note in exchange for consumption goods. The producer produces $\gamma$ goods for the banker and acquires one private note in return.
In Eq. (10), a banker in state \((0, 0)\) does not have any note in circulation and also has no reserve with the clearinghouse. Therefore, this banker is allowed to issue a new note or is ready to redeem a note issued by others. With probability \(\frac{\rho_i}{\rho_0}\), she meets with a would-be producer and issues a note in exchange for \(\gamma_b\) units of goods. With probability \(\frac{2}{\rho_0}\), a note holder wishing to consume comes up and requires redemption. But redemption will take place if and only if the issuing bank has 1 unit of reserve with the clearinghouse. In other words, note redemption would occur with probability \(\frac{\rho_i}{\rho_0}\). Then during the note redemption process, the redeeming bank in state \((0, 0)\) produces \(R_{0,0}^b\) units of goods for the note holder and acquires (and destroys) the banknote handed in. At the end of this period, the reserve balance of redeeming bank would be credit by one by the clearinghouse while the original issuer’s balance would be debited by one.

In Eq. (11), a banker in state \((0, 1)\) has 1 unit of reserve with the clearinghouse but no outstanding banknote. Due to the unity reserve limit, she cannot redeem banknotes issued by other banks. The only option for her is to purchase some consumption goods while the original issuer’s balance would be debited by one.

5.2. Stationary equilibrium

As in Section 3, we consider only symmetric stationary equilibrium. The implications of the bargaining rule that agents who produce do not gain in a trade give us the following terms of trade and incentive constraints:

\[
-q_b + V_0^a - V_0^b = -\gamma_b + V_1^a - V_1^b = 0
\]  
\[
-r_{0,0}^b + V_{0,1}^b - V_{0,0}^b = -r_{1,0}^b + V_{1,1}^b - V_{1,0}^b = 0
\]  
\[
u(q_b) + V_0^a - V_0^b \geq 0
\]  
\[
u(R_{0,0}^b) + V_0^b - V_0^a \geq 0, \quad i = 0, 1
\]  
\[
u(\gamma_b) + V_{1,0}^b - V_{0,0}^b \geq 0
\]  
\[
u(\gamma_b) + V_{1,1}^b - V_{0,1}^b \geq 0
\]  

Holders of banknote set the prices so that agent who acquires the note is just indifferent between accepting and rejecting the exchange. Eqs. (8) and (14) imply that \(V_0^a = 0\) and \(q_b = \gamma_b = V_1^a\) since holders of banknote extract the entire surplus from trades. In a likewise manner, Eqs. (12) and (15) indicate that \(V_{0,0}^b = 0\) and \(V_{1,1}^b = R_{1,1}^b\).

Let us insert the above terms of trade into the system of value functions and solve for the following:

\[
V_i^a = \frac{\rho_0^b u(\gamma_b) + \sum_{i=0}^{1} \rho_i^b u_{i,1}^b u(R_{0,0}^b)}{2r + \rho_0^b + (\rho_{0,0}^b + \rho_{1,0}^b) \rho_{1,1}^b}
\]  
\[
V_{0,0}^b = \frac{\rho_0^b u(\gamma_b)}{2r + \rho_0^b}
\]  
\[
V_{0,1}^b = \frac{\rho_0^b [u(\gamma_b) + R_{1,0}^b]}{2r + \rho_0^b}
\]  

Here, \(R_{0,0}^b\) denotes the steady-state redemption value of a bank claim set by a redeeming bank in state \((i, 0)\), where \(i = 0, 1\). According to the results in Eqs. (15), (18) and (19), we have:

\[
\frac{\rho_0^b u(\gamma_b)}{2r + \rho_0^b} + \frac{\rho_0^b [u(\gamma_b) + R_{1,0}^b]}{2r + \rho_0^b}
\]  

It implies

\[
\frac{\rho_0^b [u(\gamma_b) + R_{1,0}^b]}{2r + \rho_0^b} = \frac{\rho_0^b u(\gamma_b)}{2r + \rho_0^b}
\]
According to Eq. (20), $R_{1,0}^b$ is greater than $R_{0,0}^b$. As we know, each banker in state $(1, 0)$ has 1 unit of banknote in circulation and no reserves with the clearinghouse. In this case, they can not issue a new note unless the old one has been redeemed. In order to prepare their previously-issued notes ready for redemption and resume the banking privilege, bankers in state $(1, 0)$ must back their outstanding notes by reserves as early as possible, which could be achieved by helping redeem other bankers’ notes first. This strong incentive to build reserves will induce bankers in state $(1, 0)$ to provide a higher rate of return on a bank claim than bankers in state $(0, 0)$, i.e. $R_{1,0}^b>R_{0,0}^b$. At the end of the date, the redeeming banks in state $(1, 0)$ get a reserve reimbursement from the original issuers and step into the state $(1, 1)$ thereafter.

Substituting the above results into Eqs. (8)–(13), we obtain:

$$R_{1,0}^b = \frac{\mu_b(\rho_{n,0}^b + \rho_{b,1}^b)}{2r + \rho_b^1(\rho_{n,0}^b + \rho_{b,1}^b)} \frac{\rho_{n,0}^bu(\gamma_b)}{2r + \rho_b^1}$$

Combining Eqs. (20), (21) and (22), they are three equations that solve for $\gamma_b$ and $R_{1,0}^b$ ($i=0,1$) in terms of stationary distribution, $\rho_n^0$ ($x=n; ~i=0; ~j=0,1$). By Eqs. (20) and (21), if $r$ is sufficiently close to 0, the redemption value $R_{1,0}^b$ approaches $R_{1,0}^\beta$. Meanwhile, $R_{1,0}^\beta$ approaches $u(\gamma_b)$, where $u(\gamma_b)\geq\gamma_b$ by inequality (16). That is, when $r$ (or $\beta$) is sufficiently small (large), we have: $q\geq R_{1,0}^\beta\geq R_{1,0}^b\geq\gamma_b=(\hat{\gamma}_b)$.

Now, it only remains to determine the inventory distribution of the stationary equilibrium. Given unity banknote and reserve limit, there are

$$\rho_n^1 = \rho_{n,0}^b + \rho_{b,1}^b \tag{23}$$

$$s\mu_b = \rho_{b,1}^b + \rho_{b,1}^b \tag{24}$$

All measures must be between 0 and 1. That is

$$0<\rho_{b,1}^b<\phi\mu; \quad 0<\rho_{n,0}^b<(1-\phi)\mu_b \quad \text{and} \quad 0<\rho^b_1<1-\mu_b$$

The number of outstanding banknotes is supposed to be constant in the steady state. The flow of notes into the non-banking sector is $\rho_n^0(\rho_{n,0}^b + \rho_{b,1}^b)$ and the flow of notes out of the non-banking sector is $\rho_n^1(\rho_{n,0}^b + \rho_{b,1}^b)$. In steady state, the following must hold:

$$\rho_n^0(\rho_{n,0}^b + \rho_{b,1}^b) = \rho_n^1(\rho_{n,0}^b + \rho_{b,1}^b)$$

That is, the number of agents redeeming bank claims equals the number of new bank liability holders. In a likewise manner, the steady-state inflow to state $(0, 0)$ is $\rho_n^0(\rho_{n,0}^b + \rho_{b,1}^b)$ and outflow is $\rho_n^0(\rho_{n,0}^b + \rho_{b,1}^b)$. Equating the inflow and outflow to derive:

$$\rho_{b,0}^0 = \rho_n^0(\rho_{n,0}^b + \rho_{b,1}^b) \tag{26}$$

Combining Eqs. (24) and (25) yields:

$$\rho_{b,0}^0 = \rho_n^0(\rho_{n,0}^b + \rho_{b,1}^b) \tag{27}$$

Repeating the above procedures with bankers in other states, we can only derive either Eq. (26) or (27). No new stationary condition is found. By Eqs. (7), (21) and (24), we have:

$$R_{1,0}^b = \frac{\mu_b(1-\phi)\mu_b}{2r + \rho_b^1(1-\phi)\mu_b} \frac{(1-\mu_n-\rho_{b,1}^b)u(\gamma_b)}{2r + 1-\mu_n-\rho_{b,1}^b}$$

Note that, $R_{1,0}^b$ should locate between $\gamma_b$ and $\hat{q}$. If $R_{1,0}^b$ was smaller than $\gamma_b$, then it is preferable for holders of banknote to continue holding the note instead of getting it redeemed so that she can exchange it for goods in the future. This implies, $R_{1,0}^b>\gamma_b$. If $R_{1,0}^b$ is too large, then the redemption value of a banknote will be sufficiently attractive that nobody wants to trade away a banknote for goods. In this case, the banknote can not circulate as a medium of exchange, but only a store of value. Thus, $R_{1,0}^b<\hat{q}$.

**Definition 2.** A monetary steady-state equilibrium with banking is a vector of value functions $V=[V_n^0, V_n^1, V_b^0, V_b^1, V_{b,1}^0, V_{b,1}^1]$, quantities of trade $Q=[\gamma_b, q_b, R_{1,0}^b, i=0, 1]$ and a distribution of agents $\rho=[\rho_n^0, \rho_n^1, \rho_b^0, \rho_b^1, \rho_{b,1}^0, \rho_{b,1}^1]$ such that $\gamma_b=q_b>0$, $\hat{q}>R_{1,0}^b>R_{0,0}^b>\gamma_b$ and
where the measures of larger \( \phi \) results in either larger measure of \( \rho_{b1} \), or \( \rho_{i1} \), or both. Larger \( \rho_{b1} \) implies more bankers have reserves in stock and are prepared to issue banknotes in need. Larger \( \rho_{i1} \) implies more bankers have already issued private notes and their liabilities are backed by reserves. Then more requests for note redemption could be honored on demand. However, we need to realize a bigger welfare in the equilibrium with \( \rho_{b1} \), at the same time, might crowd out some banking transactions due to the unitary constraint on reserves and notes issue. Therefore, a higher value usually leads to a larger trade volume, which in turn, indicates a higher welfare. When \( \phi \) is extremely big, the crowding-out effect mentioned above, however, might reduce welfare. The above points are demonstrated by the following Fig. 1, where parameter values are \( r = .01, \mu_b = .6, u(q) = \alpha q \), where \( \alpha = 2 \).

Next, a numerical example is employed to compare welfare across equilibria by the criterion \( W \). By fixing the value of \( \phi \), we then consider the effects of \( \mu \) on welfare, where \( \mu \) represents the stock of money in the regime of "fiat money only" or the size of the banking sector in the regime of "private money only". In Fig. 2, parameters are chosen arbitrarily as follows: \( u(q) = \alpha q \), \( \alpha = 2 \), \( r = .01 \) and \( \phi = .5 \). Welfare in both types of equilibria rises as \( \mu \) increases up to some threshold, \( \mu^* \). When \( \mu \) is lower than \( \mu^* \), welfare in the equilibrium with "private money only" is strictly higher than that in the equilibrium with "fiat money only".\(^1\)

As we know, in the regime of "fiat money only", when a seller randomly matches a potential buyer, an exchange will take place if and only if the buyer is fortunate enough to have a unit of money. Such exchanges are all feasible in the regime of "private money only", where those two welfare curves intersect. Given \( Q \) and \( \rho \), \( V \) satisfies the system of value functions (8)–(13).

2. Given \( V, Q \) satisfies terms of trade and incentive constraints.

3. \( \rho \) satisfies Eqs. (23)–(27).

**Proposition 2.** A unique monetary steady-state equilibrium exists if private bank notes are the only circulating liabilities and the discount factor \( \beta \) is sufficiently close to one.

5.3. Welfare analysis

To evaluate the welfare in this economy, we use a generalized welfare criterion of average expected utility across the population in the steady state. We let \( W \) denote the aggregate welfare, then

\[
W = \rho_{m} V_{m}^{n} + \rho_{b} V_{b}^{n} + \rho_{i} V_{i}^{n} + \sum_{i=0}^{1} \sum_{j=0}^{1} \rho_{ij} V_{ij}^{b}
\]

where the measures of \( \rho_{i} \) and \( \rho_{ij} \) \((i = 0, 1; j = 0, 1)\) are zero in the regime of "fiat money only", while in the regime of "private money only", the measure of \( \rho_{m}^{b} \) is zero.

Due to the complexity of the model, we use numerical examples for illustration. First, consider the effects of \( \phi \) on welfare in the regime of "private money only", where \( \phi \) denotes the measure of bankers who are randomly endowed with reserves. Given \( \mu_b \), larger \( \phi \) results in either larger measure of \( \rho_{b1} \), or \( \rho_{i1} \), or both. Larger \( \rho_{b1} \) implies more bankers have reserves in stock and are prepared to issue banknotes in need. Larger \( \rho_{i1} \) implies more bankers have already issued private notes and their liabilities are backed by reserves. Then more requests for note redemption could be honored on demand. However, we need to realize a bigger welfare in the equilibrium with \( \rho_{b1} \), at the same time, might crowd out some banking transactions due to the unitary constraint on reserves and notes issue. Therefore, a higher \( \phi \) value usually leads to a larger trade volume, which in turn, indicates a higher welfare. When \( \phi \) is extremely big, the crowding-out effect mentioned above, however, might reduce welfare. The above points are demonstrated by the following Fig. 1, where parameter values are \( r = .01, \mu_b = .6, u(q) = \alpha q \), where \( \alpha = 2 \).

As we know, in the regime of "fiat money only", when a seller randomly matches a potential buyer, an exchange will take place if and only if the buyer is fortunate enough to have a unit of money. Such exchanges are all feasible in the regime of "private money only", where those two welfare curves intersect. Given \( Q \) and \( \rho \), \( V \) satisfies the system of value functions (8)–(13).

1. Given \( Q \) and \( \rho \), \( V \) satisfies the system of value functions (8)–(13).

2. Given \( V, Q \) satisfies terms of trade and incentive constraints.

3. \( \rho \) satisfies Eqs. (23)–(27).

**Proposition 2.** A unique monetary steady-state equilibrium exists if private bank notes are the only circulating liabilities and the discount factor \( \beta \) is sufficiently close to one.

5.3. Welfare analysis

To evaluate the welfare in this economy, we use a generalized welfare criterion of average expected utility across the population in the steady state. We let \( W \) denote the aggregate welfare, then

\[
W = \rho_{m} V_{m}^{n} + \rho_{b} V_{b}^{n} + \rho_{i} V_{i}^{n} + \sum_{i=0}^{1} \sum_{j=0}^{1} \rho_{ij} V_{ij}^{b}
\]

where the measures of \( \rho_{i} \) and \( \rho_{ij} \) \((i = 0, 1; j = 0, 1)\) are zero in the regime of "fiat money only", while in the regime of "private money only", the measure of \( \rho_{m}^{b} \) is zero.

Due to the complexity of the model, we use numerical examples for illustration. First, consider the effects of \( \phi \) on welfare in the regime of "private money only", where \( \phi \) denotes the measure of bankers who are randomly endowed with reserves. Given \( \mu_b \), larger \( \phi \) results in either larger measure of \( \rho_{b1} \), or \( \rho_{i1} \), or both. Larger \( \rho_{b1} \) implies more bankers have reserves in stock and are prepared to issue banknotes in need. Larger \( \rho_{i1} \) implies more bankers have already issued private notes and their liabilities are backed by reserves. Then more requests for note redemption could be honored on demand. However, we need to realize a bigger welfare in the equilibrium with \( \rho_{b1} \), at the same time, might crowd out some banking transactions due to the unitary constraint on reserves and notes issue. Therefore, a higher \( \phi \) value usually leads to a larger trade volume, which in turn, indicates a higher welfare. When \( \phi \) is extremely big, the crowding-out effect mentioned above, however, might reduce welfare. The above points are demonstrated by the following Fig. 1, where parameter values are \( r = .01, \mu_b = .6, u(q) = \alpha q \), where \( \alpha = 2 \).

Next, a numerical example is employed to compare welfare across equilibria by the criterion \( W \). By fixing the value of \( \phi \), we then consider the effects of \( \mu \) on welfare, where \( \mu \) represents the stock of money in the regime of "fiat money only" or the size of the banking sector in the regime of "private money only". In Fig. 2, parameters are chosen arbitrarily as follows: \( u(q) = \alpha q \), \( \alpha = 2 \), \( r = .01 \) and \( \phi = .5 \). Welfare in both types of equilibria rises as \( \mu \) increases up to some threshold, \( \mu^* \). When \( \mu \) is lower than \( \mu^* \), welfare in the equilibrium with "private money only" is strictly higher than that in the equilibrium with "fiat money only".\(^1\)

As we know, in the regime of "fiat money only", when a seller randomly matches a potential buyer, an exchange will take place if and only if the buyer is fortunate enough to have a unit of money. Such exchanges are all feasible in the regime of "private money only".
only”. Moreover, private note creates the possibility for trade that will not happen in a fiat money regime. For instance, when a banker meets a non-banker seller where the former does not have money, trade may still continue since the banker might be able to issue a unit of banknote in exchange for consumption goods. Also, production occurs during the process of note redemption. Therefore, the expected lifetime utility in the regime of “private money only” is supposed to be relatively higher due to higher frequency of trade. At the same time, a banknote holder in this model can redeem a note anywhere at par through the clearinghouse. The related discounts or premia on banknotes disappear and thereby welfare is improved. When \( \beta \) approaches 1, holders of banknotes do not face substantially different rates of return on their assets. Therefore, the lowered redemption cost and equality in rates of returns result in higher welfare.

When \( \mu \) is extremely large, particularly \( \mu > \bar{\mu} \), welfare in both equilibria begins to fall. The regime of “fiat money only” holds higher welfare than that in “private money only”. An increase in \( \mu \) introduces more medium of exchange but squeezes producers out of the economy in both equilibria. When the loss in reducing production outweighs the gain in overcoming trade frictions, increasing \( \mu \) reduces welfare, but it cuts welfare more in the regime of “private money only”.

As the size of the banking sector grows larger, the size of non-banking sector becomes smaller. So does the measure of would-be producers. Situation is now difficult for both bankers and non-bankers. First, it is hard for bankers to issue banknotes due to the rapid reduction in producers (note recipients). Therefore, the measure of \( \rho_0^B \) and \( \rho_0^C \) accumulates, while the measure of \( \rho_1^B \) and \( \rho_1^C \) decreases. This implies that the quantity of banknotes in circulation falls as \( \mu \) grows. Second, the shrinking measure of producers also markedly discourages trades between holders of banknotes and producers. Some note holders who wish to consume might turn to bankers in state \((0, 0)\) or \((1, 0)\) for note redemption. However, not many redemption requests would be honored due to the tiny measure of \( \rho_1^B \) at present. That is, few bankers are ready to have their issued notes redeemed. So, an extremely large banking sector \((\mu > \bar{\mu})\) would result in fewer producers altogether with scarcer medium of exchange (banknotes) in the regime of “private money only”. This causes a sharp decrease in welfare accordingly.

On the other hand, in the regime of “fiat money only”, when \( \mu > \bar{\mu} \), increasing the stock of money does crowd out production, but the medium of exchange (fiat money) is still plentiful. In this case, welfare does decrease but not as much as it otherwise would have in the former regime. Welfare is, therefore, higher than that in the regime of “private money only”.

**Proposition 3.** If the discount factor \( \beta \) is sufficiently large and the size of the banking sector \((\mu_b)\) is properly specified (for instance \( \mu_b < \bar{\mu} \)), then the production and welfare in a monetary steady state with “private money only” is strictly higher than that attained by a steady state in which banks are prohibited from issuing private notes.

Before proceeding to the next section, we would like to point out some feature of this model. In the regime of “private money only”, when \( \mu > \bar{\mu} \), there are “less production and fewer banknotes”. It implies the bargaining power of note holders will not be affected a lot in transactions and the price of goods remains comparatively stable. By contrast, in the regime of “fiat money only”, when \( \mu > \bar{\mu} \), there are “less production but more money”, which of course will disadvantage holders of money in bilateral bargaining and therefore drive up the price of goods accordingly.\(^2\) In other words, the over-issuing problem leads to an obvious

\(^2\) In the regime of “fiat money only”, the price of goods is defined as \( \text{The amount of fiat money} \). Similarly in “private money only” the price of goods is \( \text{The amount of goods purchased} \).
increase in the price of goods in the regime of “fiat money only”. Therefore, private monetary system is more self-regulating in supplying currency, which lends support to the result obtained by Cavalcanti et al. (1999).

6. Coexistence of fiat money and private money

In Sections 3 and 5, we have explored the economic implications of fiat money and private notes, respectively. Under certain conditions, the existence of safe and privately-issued liabilities does improve welfare. Now, we will move forward to investigate the monetary system where these two types of assets coexist.

6.1. Economic environment

A fraction \( \mu_b \) of agents start with fiat money and a fraction \( \mu_n (0 < \mu_m + \mu_b < 1) \) begin with clearinghouse membership (i.e. bankers). As in Section 5, a fraction \( \phi \) of bankers \((\phi \cdot \mu_b) \) is randomly chosen to have 1 unit of indivisible reserve deposit with the clearinghouse in terms of some outside money. The remaining agents \((1 - \mu_n - \mu_b) \) are non-bankers without fiat money, who consist of would-be producers and holders of banknotes.

The sequence of actions within a period occurs as follows. At the beginning of a period, the preference shock is revealed. A banker who wishes to consume but can not issue a new note is allowed to convert her central reserve into one unit of fiat money through the clearinghouse. Assume she chooses to implement this conversion with probability \( \lambda_1 (0 \leq \lambda_1 \leq 1) \) and then she is able to purchase consumption goods with fiat money in the following random meetings. To simplify matters, her reserve balance at the clearinghouse keeps unchanged until the end of the day but will be frozen and marked as “unavailable for redemption”. Each banker could contact the clearinghouse costlessly to acquire necessary information.

Next, agents randomly match and begin bargaining. To facilitate note redemption, an individual banker who has already issued a note but had no reserves with the clearinghouse is allowed to increase her reserves by producing goods in exchange for fiat money with probability \( \lambda_0 (0 \leq \lambda_0 \leq 1) \). If a trade does happen, the banker could deposit her monetary proceeds with the clearinghouse as early as possible. At the end of this period, the clearinghouse accomplishes the reserve settlement and updates the reserve balance for all bankers. Then a new period starts.

We will still restrict attention to symmetric steady states throughout this section. At the end of a period, an agent can be in one of seven possible states, depending on her identity (banker or non-banker) and the monetary object that is held in inventory. Let \( V_{n,f}^b (n = 0, 1; f = 0, 1) \) be the value of bankers in a corresponding state, \( V_{n,f}^m \) the value of non-bankers holding a banknote, \( V_{n,f}^b \) the value of holding a unit of fiat money with probability \( \lambda_1 \), and \( V_{n,f}^m \) the value of holding nothing. We let \( q_b \) and \( q_m \) be the price at which a bank note or a unit of fiat money trades among non-bankers in the steady state, respectively. Let \( q_{b,b} \) denote the trading price between a banker and a holder of fiat money. As in Section 5, \( q_b \) and \( q_{b,b} \) can be interpreted in a likewise manner.

The inventory distribution is described by \( \rho_m^b, \rho_b^b, \rho_b^b, \rho_0^b, \rho_0^b, \rho_{1,0}^b, \rho_{1,1}^b \), where \( \rho_m^b \) is the fraction of money holders, \( \rho_b^b \) is the fraction of note holders, \( \rho_0^b \) is the measure of producers and other \( \rho_{i,j}^b \) \((i = 0, 1; j = 0, 1) \) are defined similarly as in Section 5. Then, we have

\[
\rho_0^b + \rho_1^b = 1 - \rho_m^b - \mu_b
\]

We assume that no exchanges take place between agents who are both holding assets. Agents choose production, consumption and trading strategies in order to maximize the expected discounted utility of consumption net of production costs. The Bellman equations, in flow terms, are

\[
rV_{1,0}^b = \frac{\lambda_0 \rho_m^b}{2} \left[ -q_m^b + V_{1,1}^b - V_{1,0}^b \right] + \rho^b_{1,1} \left( -q_1^b + V_{1,0}^b + V_{1,1}^b - V_{1,0}^b \right) - \lambda_1 \rho_1^b \left( -q_1^b + V_{1,0}^b + V_{1,1}^b - V_{1,0}^b \right)
\]

\[
rV_{1,1}^b = \frac{\lambda_0 \rho_m^b}{2} \left[ q_m^b + V_{1,1}^b + V_{1,0}^b \right] + \rho^b_{1,1} \left( -q_1^b + V_{1,0}^b + V_{1,1}^b - V_{1,0}^b \right) - \lambda_1 \rho_1^b \left( -q_1^b + V_{1,0}^b + V_{1,1}^b - V_{1,0}^b \right)
\]

\[
rV_{0,1}^b = \frac{\lambda_0 \rho_m^b}{2} \left[ q_m^b + V_{1,1}^b - V_{1,0}^b \right] + \rho^b_{1,1} \left( -q_1^b + V_{1,0}^b + V_{1,1}^b - V_{1,0}^b \right) - \lambda_1 \rho_1^b \left( -q_1^b + V_{1,0}^b + V_{1,1}^b - V_{1,0}^b \right)
\]

\[
rV_{0,0}^b = \frac{\lambda_0 \rho_m^b}{2} \left[ q_m^b + V_{1,0}^b - V_{1,0}^b \right] + \rho^b_{1,1} \left( -q_1^b + V_{1,0}^b + V_{1,1}^b - V_{1,0}^b \right) - \lambda_1 \rho_1^b \left( -q_1^b + V_{1,0}^b + V_{1,1}^b - V_{1,0}^b \right)
\]

\[
rV_{0}^b = \frac{\lambda_0 \rho_m^b}{2} \left[ -q_m^b + V_{1,0}^b - V_{1,0}^b \right] + \rho_1^b \left( -q_1^b + V_{1,0}^b + V_{1,1}^b - V_{1,0}^b \right) - \lambda_1 \rho_1^b \left( -q_1^b + V_{1,0}^b + V_{1,1}^b - V_{1,0}^b \right)
\]

\[
rV_{m}^b = \frac{\lambda_0 \rho_m^b}{2} \left[ -q_m^b + V_{1,0}^b - V_{1,0}^b \right] + \rho_1^b \left( -q_1^b + V_{1,0}^b + V_{1,1}^b - V_{1,0}^b \right) - \lambda_1 \rho_1^b \left( -q_1^b + V_{1,0}^b + V_{1,1}^b - V_{1,0}^b \right)
\]
In Eq. (31), the value of being bankers in state (1, 0) at the end of the current period is determined by the opportunities of trading in the following period. These opportunities need to be discounted to the present using the rate of time preference \( r \). First, in the following period, with probability \( \rho_m^b \), this banker might meet an agent with a unit of fiat money, trade can only occur if she is willing to trade and the other agent wishes to consume, which happens with probability \( \frac{\rho}{2} \). If there is a trade, then the banker produces \( q^b_m \) goods in exchange for the fiat money. At the end of this period, she deposits the monetary proceeds with the clearinghouse as a unit of reserve. Then her state evolves into (1, 1). The second term describes the situation where a holder of banknote comes up and asks for note redemption. This requirement will be honored conditional on the original issuing bank currently has 1 unit of “unfrozen” reserve with the clearinghouse, which occurs with probability \( \rho^b_{1,1} \left[ \frac{1}{2} + \frac{1}{2} (1 - \lambda_1) \right] = \rho^b_{1,1} \frac{1}{2} (1 - \lambda_1) \). During the process of note redemption, \( R^b_{1,0} \) goods are produced by the redeeming banker and her reserve balance at the clearinghouse would be reimbursed by 1 at the end of the day.

Eq. (32) describes the flow value of a banker in state (1, 1). In a given period, with probability \( \frac{1}{2} \), the banker wishes to consume \( (\theta_1 = 1) \). Driven by the desire for consumption, she converts her central reserve into a unit of fiat money with probability \( \lambda_1 \). With a probability \( \frac{1}{2} \), the preference shock \( \theta_0 \) is 0 and she does not like consumption today. In this case, she would keep her reserve in stock with the clearinghouse and make her previously-issued note ready for redemption. All other equations could be interpreted similarly.

6.2. Participation constraints

Given take-it-or-leave-it offers by holders of assets, the following conditions must hold:

\[-q_m + V^b_{m0} - V^b_0 = 0 \]
\[-q_b + V^b_{10} - V^b_0 = -\gamma_b + V^b_0 - V^b_0 = 0 \]
\[-R^b_{i,0} + V^b_{i,1} - V^b_{i,0} = 0, \quad i = 0, 1 \]
\[-q^b_m + V^b_{i,1} - V^b_{i,0} = 0 \]
\[u(q_m) + V^b_{m0} - V^b_m \geq 0 \]
\[u(q_b) + V^b_{10} - V^b_1 \geq 0 \]
\[u \left( R^b_{i,0} \right) + V^b_{i} - V^b_{i,0} \geq 0, \quad i = 0, 1 \]
\[u(\gamma_b) + V^b_{i,1} - V^b_{i,0} \geq 0, \quad i = 0, 1 \]

The bargaining rule implies that producers get zero surplus while consumers would extract all the positive surplus from bilateral exchanges. Therefore, we have

\[V^b_{00} = V^b_{10} = 0, \quad V^b_{m0} = q_m \quad \text{and} \quad V^b_{1} = q_b \quad \text{(33)} \]
\[q_b = \gamma_b \quad \text{(34)} \]
\[q^b_m = R^b_{1,0} \quad \text{(35)} \]

Eq. (35) implies bankers in state (1, 0) are just indifferent between trading with holders of banknote and holders of fiat money. Either will cost them the same and improve their reserve by 1 unit, which indicates the probability \( \lambda_0 \) can be set to 1. The system of value functions and participation constraints suggest that as the rate of time preference, \( r \), gets sufficiently close to zero, we can write

\[\hat{q} > R^b_{1,0} \geq R^b_{0,0} \geq \gamma_b = q_b \]

\[R^b_{1,0} \geq q_m \geq q_b = \gamma_b \]
That is, to attract patient agents to accept the intrinsically useless pieces of paper, bankers have to promise a redemption value \( R_{i0} \) \((i = 0, 1)\) which is no less than \( \gamma_b \), the cost to acquire a banknote. Moreover, fiat money in this model shall be traded at a price \( q_{m0} \) which is at least as big as \( q_b \), the trading price of banknotes among non-bankers, because fiat money here not only circulates as a medium of exchange but also can be directly used as a reserve asset. Therefore, the relationship among \( R_{i0} \), \( q_{m0} \), and \( q_b \) suggests when the preference shock \( \theta_t = 1 \) bankers in state \((1, 1)\) would be at least indifferent between converting reserves into fiat money and holding it for expected note redemption to regain banking privilege. It indicates the probability \( \lambda_1 \) can be reasonably set to 1 also.

### 6.3. Stationary equilibrium

The stationarity across states implies that the inflow into a particular state must equal the outflow from this state for both bankers and non-bankers. Using the same logic as in Section 5 and substituting \( \lambda_0 = \lambda_1 = 1 \) into value functions, we obtain

\[
\rho_{m0}^{b1,0} = \rho_{0}^{b1,1} \tag{36}
\]

\[
\rho_{0,0}^{b1,0} = \frac{1}{2} \rho_{1,0}^{b1,1} \tag{37}
\]

\[
\rho_{0,1}^{b1,0} = \frac{1}{2} \rho_{1,0}^{b1,1} \tag{38}
\]

The remaining conditions the stationary distribution has to satisfy are described by Eqs. (23), (24) and (30).

**Definition 3.** A monetary steady-state equilibrium with coexistence of fiat money and private notes is a vector of value functions \([V_0^m, V_1^m, V_0^b, V_1^b]\), quantities of trade \(Q = [q_{m0}, q_b, q_{m0}, R_{i0}, R_{i0}]\), \(i = 0, 1\) and a distribution of agents \(\rho = [\rho_{m0}^{b1,1}, \rho_{0}^{b1,1}, \rho_{0,0}^{b1,1}, \rho_{0,1}^{b1,1}]\) such that \(q_{m0} > 0, \gamma_b > q_b, q > R_{10}^{10} > R_{00} > \gamma_b\) and

1. Given \(Q\) and \(\rho\), \(V\) satisfies the system of value functions.
2. Given \(V\), \(Q\) satisfies terms of trade and incentive constraints.
3. \(\rho\) satisfies conditions (23), (24), (30) and (36)–(38).

**Proposition 4.** Two steady-state equilibria may exist due to strategic complementarity. In one of these, fiat money is more favored, and in the other the both private notes and fiat money are equally valued.

Only when the size of the banking sector \(\mu_b\) is sufficiently large, or banknotes dominate in transactions, will fiat money play a role of reserves. In this case, as more banknotes circulate, fiat money is more needed by banks in building up reserves. Therefore, money can circulate at a price no less than \(q_b\) and get a non-negative return. That is, “coexistence” makes fiat money more valuable than when it circulates alone. However, \(\mu_b\) cannot be too large, or else monetary assets are not valued in the equilibrium any more, as analyzed in the regime of “private money only”. On the other hand, if \(\mu_b\) is getting close to 0, then the average time lag between

![Fig. 3. Existence of stationary equilibrium.](image-url)
note issue and note redemption is prolonged accordingly, which will depress the issue of private notes but promote the circulation of fiat money in the economy. Thus, given $\phi$, to support valid stationary equilibria in the regime of "coexistence", the value of $\mu_b$ must have a reasonable range. For illustration, consider an example where we fix parameter values, excluding $\mu_b$ and $\phi$, and then determine the regions of the parameter values ($\phi$, $\mu_b$) for which the monetary steady-state equilibria with banking exist. In Fig. 3, $u(q) = aq$, $\alpha = 2$, $r = .01$ and $m = .05$.

The proof of Proposition 4 shows for a certain quantity of trade ($Q$), there exist two possible inventory measures ($\rho$), which satisfy all the constraints. Therefore, multiple equilibria occur. When the banking sector is sufficiently enlarged, the quantity of circulating private notes increases, which makes fiat money more valuable as reserves to back the outstanding private liabilities. Due to the different rates of return on fiat money and private notes, people show preference over assets. The decision about what asset to accept during the trade is affected by how valuable this asset is and how easy it is to trade in the future. This in turn is determined by how many would-be producers and bankers there are, which is determined by what asset is accepted by other agents. Thus, strategic complementarity introduces the possibility of multiple steady-state equilibria, which can be welfare-ranked.

6.4. Welfare analysis

In this section, welfare in the money and banking equilibrium is also defined by Eq. (29). To demonstrate the welfare comparison across these three monetary regimes, the following Fig. 4 is drawn by fixing the values of $r$, $m$ and $\phi$. Specifically, $u(q) = aq$, $\alpha = 2$, $r = .01$, $\phi = .5$ and $m = .05$.

Shown in Fig. 4, when $\mu_b \leq \hat{\mu}$, a suboptimal quantity of private notes is produced. In this case, people just do not care what kind of assets they are holding. As a medium of exchange, both fiat money and banknotes are treated the same. Generally, more monetary assets indicate higher welfare. However, when the size of the banking sector is sufficiently expanded (i.e. $\hat{\mu} < \mu_b \leq \mu$) the quantity of circulating private notes is also increased, which makes fiat money more valuable as reserves to back the outstanding private liabilities. Multiple equilibria occur due to the strategic complementarity inherent in the diversification in intermediation.

Specifically, if people prefer monetary exchanges over banknote exchanges, producers would like to set higher reservation value in trade meetings since they are waiting for possible monetary trades. However, their waiting may be just in vain because the quantity of available fiat money is limited. Moreover, money holders are more willing to trade with bankers for $R_{1,0}^b$ goods instead of trading with producers for $q_m$ goods, where usually $R_{1,0}^b \geq q_m$. So, this "preference for monetary exchanges" accumulates the measure of producers and decreases circulating banknotes (lower $\rho_{1,0}^b$ and $\rho_{1,1}^b$), which results in fewer exchanges and less production in the economy. On the contrary, if people do not have special favor on monetary exchanges, banknotes turn out to be perfect substitutes, which stimulates the development of the banking sector. The issue of banknotes will be greatly encouraged.

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3 According to Temin (1969), almost 90% of the U.S. money supply in 1830 consisted of private banknotes. This is the reason that $m = .05$ is chosen for the following numerical illustration. Other values of $m$ also have been used and we got similar results showed by Figs. 3 and 4.

4 Here, $\bar{\mu}$ is the upper bound of $\mu_b$, in the regime of "coexistence", i.e. $0 < \mu_b < \bar{\mu}$. In Fig. 4, $\bar{\mu} \approx .462$ and $\mu \approx .645$. 
This increases the float of private money (higher $\rho_{1,0}$ and $\rho_{1,1}$) and creates more trading opportunities. That is, when $\bar{\mu} - \mu_b \leq \tilde{\mu}$, the equilibrium "without a preference over assets" (gray dotted line) dominates that "with a preference "(gray solid line) in welfare terms, as shown in Fig. 4.

If agents could somehow choose among steady states, they would all agree on which one was most preferred. However, the problem is that, in decentralized search equilibrium, the outcome may not be this most-preferred steady state due to the assumed private trading histories of non-bankers and the lack of communication between the clearinghouse and non-bankers. This is what we called "coordination problem" discussed by Diamond (1982). To avoid the coordination failure, we require either free flow of information among agents or intervention from coordinating institutions, such as financial intermediaries. It is possible though, if agents could get together and communicate freely, the bad outcome like the welfare-dominated steady state studied here would not occur at all. However, in practice, it is certainly difficult for all economic agents to communicate perfectly. At the same time, inefficient intervention by coordinating institutions may not be able to eliminate the bad steady state and it might even make the good one worse. Then finally, we might end up with a compromise—the steady state with the medium welfare in the regime of "private money only".

7. Conclusion

A random-matching model incorporated with a clearinghouse was constructed to analyze the social welfare associated with three different monetary arrangements, which vary according to the asset circulating in the economy.

Fiat money was first restricted to be the only monetary asset in the economy. Under this inelastic currency regime, transactions will not occur unless the potential buyer carries one unit of money. Therefore, any monetary arrangement that removes this constraint will improve welfare. This can be done by permitting banking arrangements and circulating private notes. Then we showed that economy with "private money only" possess a unique steady state as the regime of "fiat money only" does. If the size of the banking sector is not extremely large, welfare is comparatively higher than what is achieved in the former regime because of higher frequency of trades. In economies with both circulating private liabilities and government-issued fiat money, two monetary steady states appear due to strategic complementarity. In one steady state, "preference for monetary exchanges" suppresses the issue of bank notes, which leads to lower welfare. In the other steady state, the public was largely indifferent between holding bank notes and lawful money, which stimulates the development of banking sector and achieve higher welfare. These two steady states, which are ranked in terms of welfare, represent a coordination problem.

Our results lend support to the notion that the presence of safe and privately-issued liabilities is not necessarily the origin of "multiplicity of equilibria" or "endogenous volatility". The introduction of fiat money can improve matters only when the private banking sector produces a suboptimal quantity of medium of exchange. Otherwise, a monetary arrangement with private money is more welfare-improving.

There are, of course, many important issues regarding private money that our analysis has not addressed. For example, there can be private information concerning the quality of banknotes, which gives rise to a lemons problem. Then it would be interesting to reconsider which monetary arrangement is preferable under private information, "fiat money only", "private money only" or "coexistence of two assets". Also, the size of the banking sector, $\mu_b$, can be endogenized. That is, by setting up some policy parameter, we allow agents to choose between being a banker and a non-banker. Then we will examine how the size of the banking sector and the corresponding welfare are affected when the value of that policy parameter changes. We hope to explore all these in our future research.

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Appendix A

A sufficiently large $\beta$ is essential for the existence of monetary equilibria. As the discount factor $\beta$ increases and so does the reward for future consumption. Agents are willing to accept money as long as money remains relatively scarce.

Proof of Proposition 1

In order to demonstrate the existence of a stationary monetary equilibrium, we solve the value functions to get

\[ V_m = \frac{\rho_0 u(q_m)}{2r + \rho_0} \]

\[ V_0 = 0 \]
Because $u(\cdot)$ is concave and $V_{m}^{0} = \frac{\partial u(\theta)}{\partial \theta} = q_{m}$, there exists a unique equilibrium value $\hat{q}$, where $\hat{q} < q$. Moreover, the incentive constraints obviously hold for any $\rho_{m} \in (0, 1)$.

**Proof of Proposition 2**

By Eq. (22), we can write

$$
\gamma_{b} = \frac{\rho_{b}^{0} u(\gamma_{b}) + \rho_{b1}^{0} u(R_{b1}^{b}) + \rho_{b1}^{1} u(R_{b1}^{b})}{2 + \rho_{b}^{0} + \rho_{b1}^{1}(\rho_{b1}^{0} + \rho_{b1}^{b})}
$$

where $R_{bi}^{b}$ $(i = 1, 0)$ are specified by Eqs. (20) and (21). Due to the concavity of $u(\cdot)$, there exists a unique equilibrium value $\gamma_{b}$ to the above equality, where $\gamma_{b} < \hat{q}$. Once $\gamma_{b}$ is found, other quantities of trade such as $R_{b1}^{b}$ $(i = 1, 0)$ are able to be determined. The incentive constraints hold for the distribution of agents $\rho_{b}$ that satisfies conditions (23)–(27). We actually could solve the measure of each type of agents in this unique stationary monetary equilibrium with banking. That is

$$
\rho_{b1}^{b} = \frac{1 + \mu_{b} [\phi + \mu_{b} (5 + 2 \times (-3 + \phi))]}{3 \mu_{b} (2 - \phi)} + P_{0}(\mu_{b}, \phi)
$$

Note $P_{0}(\mu_{b}, \phi)$ is a function in terms of $\mu_{b}$ and $\phi$, which was solved analytically by Mathematica. It is a very complex expression though.

To ensure the existence of an equilibrium where private notes circulate, the size of the banking sector $\mu_{b}$ cannot be too large or too small. Intuitively, if $\mu_{b}$ is too small, then the existing quantity of the medium of exchange would be so scarce that the holders of private money shall demand a skyrocketing price for each note, however the corresponding producers may not agree to supply that much. Also, it might take a long time for a note to be returned to the banking system for redemption due to the tiny measure of the banking sector. Before that, the issuing bankers can not consume. Considering this fact, bankers are reluctant to issue private notes and would rather prefer autarky, which will shrink the quantity of banknotes in circulation further more. So, a monetary equilibrium may not exist when $\mu_{b}$ is too small.

On the other hand, a private monetary system with too many issuers of banknotes (i.e. $\mu_{b}$ is too large) might also imply non-existence of monetary equilibrium. Intuitively, if everyone was endowed with the banking privilege and trades between bankers were allowed, then a gift giving system of exchange--akin to credit--could be organized without the use of money or banknotes at all. Therefore, to make the media of exchange valued at an equilibrium, the measure of $\mu_{b}$ can not be extremely large and the government should frame some appropriate standards to regulate the size of the banking sector. For instance, the government could set some cost of entering the banking sector and then $\mu_{b}$ might be controlled efficiently by adjusting that cost. Therefore, we see neither extremely low value nor extremely high values of $\mu_{b}$ favors the existence of a monetary equilibrium.

**Proof of Proposition 3**

By Eqs. (4)–(6), welfare in the equilibrium with “flat money only”, $W_{f}$, can be written as

$$
W_{f} = \frac{\mu (1 - \mu) u(q_{m})}{2 + 1 - \mu}
$$
In a likewise manner, welfare in the equilibrium with “private money only”, \( W_p \), is

\[
W_p = \sum_{i=0}^{\infty} \rho^i_0 V^i_0 = \left( \rho^0_1 + \rho^0_0 \right) \gamma_0 + \rho^1_0 \left( \gamma_0 + R^1_0 \right) + \rho^0_0 \left( \frac{1}{2} - \rho^0_1 \right) u(\gamma_0) + \rho^1_0 \left( \frac{1}{2} - \rho^0_1 \right) \left[ u(\gamma_0) + R^0_1 \right]
\]

where \( \rho^i_0 \) and \( \rho^i_1 \) (\( i = 0, 1 \) and \( j = 0, 1 \)) can be solved analytically as in Proof of Proposition 2. Since \( u(\cdot) \) is concave, the two welfare functions are concave in terms of \( \mu \). Equating \( W_f \) and \( W_p \) allows us to solve \( \hat{\mu} \) with the aid of Mathematica. The solution exists. Let \( \mu^*_b \) and \( \mu^*_m \) denote the upper limit of \( \mu_b \) and \( \mu_m \), respectively. That is, \( \mu^*_b = (0, \mu^*_m) \) and \( \mu^*_m = (0, \mu^*_m) \). By conditions (4), (7) and (23)–(27)), \( \mu_m = 1 - \rho^0_0 \) and \( \mu_b = 1 - \rho^0_0 - \rho^0_1 \). Since \( \mu_b \) is more strictly constrained, naturally \( \mu^*_b < \mu^*_m \). The non-zero solution \( \hat{\mu} \), therefore, is unique. When \( \mu < \hat{\mu} \), \( W_p > W_f \) for given quantities of trade. This implies \( W_p > W_f \). Similarly, we can show \( W_p < W_f \) for \( \mu < \hat{\mu} \).

**Proof of Proposition 4**

In the equilibrium we are examining, \( q_m = V^*_m > 0 \). Holding fiat money has strictly positive value, while acquiring a bank note implies net expected utility \( V_f - \gamma_0 = \gamma_0 - \gamma_0 = 0 \). Thus, in a steady state, no one would want to dispose of fiat money balances to acquire a bank note. Therefore, these two monetary assets can circulate together and \( \rho^i_0 = \rho^i_0 = \mu_m \) in equilibrium. That is, the fraction of money holders in the steady state is equal to the quantity of money injected by the central bank at the initial date. Taking \( \mu_m \) as an exogenous parameter and using Eqs. (7), (23), (24), (30) and (36)–(38), we can solve for the measure of each type of agent in terms of \( \mu_b, \mu_m \), and \( \phi \).

If \( \mu_b < \hat{\mu} \),

\[
\rho^0_1 = \frac{\mu_b \left( -2 - \mu_m \left( -2 + \phi \right) + \phi + \mu_b \left( 7 - \phi \left( -7 + 2 \phi \right) \right) \right)}{3 \mu_b \left( 3 - \phi \right) - 3} + P_1(\mu_b, \phi)
\]

If \( \mu_b > \hat{\mu} \),

\[
\rho^0_1 = \frac{\mu_b \left( -2 - \mu_m \left( -2 + \phi \right) + \phi + \mu_b \left( 7 - \phi \left( -7 + 2 \phi \right) \right) \right)}{3 \mu_b \left( 3 - \phi \right) - 3} + P_1(\mu_b, \phi)
\]

As in Proof of Proposition 2, \( P_i(\mu_b, \phi) \) (\( i = 1, 2 \)) and \( \hat{\mu} \) are solved analytically by Mathematica and \( P_1(\mu_b, \phi) \neq P_2(\mu_b, \phi) \). The relationship between \( \rho^i_0 \) and other \( \rho^i_j \) (\( x = n, b; i = 0, 1; j = 0, 1 \)) are the same as those embedded in Proof of Proposition 2, except that in this environment,

\[
\rho^0_0 = \frac{\mu_m \rho^1_0}{\phi \mu_b - \left( 1 - \phi \mu_b \right) \rho^1_0}
\]

When fiat money and banknotes coexist, as the size of the banking sector increases to a certain threshold \( \hat{\mu} \), multiple equilibria occur. It seems such kind of coexistence leads to multiplicity or “indeterminacy” in equilibria which is not observed in the previous two regimes.

**References**


